Modeling Moderated Variance Ratios in Mplus

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Introduction

The Mplus package (Muthén and Muthén, 2011) allows restrictions to be imposed on variables that are functions of variances and covariances. Thus, for a categorical moderator, the equal variances ratio (EVR) test described in Smithson (2012) can be incorporated into a multi-groups structural equations model (SEM) via appropriate model comparisons. These SEMs also can incorporate tests of homogeneity of (error) variance and, of course, moderation of correlations and moderation of slopes.

EVR models are implemented in correlation SEMs of the kind described in Smithson (2012). The EVR component is initiated by defining the relevant variance ratios. Suppose the variance ratios of interest are for the i^{th} and j^{th} categories of the moderator. These variance ratios must first be defined in the MODEL CONSTRAINTS statement. We can then compare a model that leaves them unconstrained against a model that imposes equality between them (via a statement such as `ratioi = ratioj`). Clearly multiple hypotheses of this kind can be tested simultaneously. Model comparisons can be made in the usual fashion.

Homogeneity of variance also can be tested in the correlation SEMs, either for x or y alone (which is incompatible with EVR) or simultaneously (which imposes EVR). Regression SEMs can include tests for homogeneity of error variance, and regression SEMs also can incorporate EVR tests. All of these capabilities are demonstrated in the examples that follow.

Examples of EVR-testing SEMs in Mplus

Two-Category Moderator

This is a two-category moderator example in which the null hypothesis of EVR is true. It uses an artificial data-set sampled from two bivariate normal distributions, associated with one category of a binary moderator variable z which takes values -1 and +1. For the first moderator category $\sigma_x^2 = 1$ and $\sigma_y^2 = 2$, while for the second category $\sigma_x^2 = 4$ and $\sigma_y^2 = 8$. Thus, the population variance ratio in both moderator categories is 1/2. The population covariances are 1 and the means are 0 for both categories. The data-file is eqdat.dat. There are 500 observations in each category. In the first category, $s_x^2 = 0.883$ and $s_y^2 = 1.958$, so $s_x^2/s_y^2 = 0.451$. In the second category, $s_x^2 = 4.068$ and $s_y^2 = 7.515$, so $s_x^2/s_y^2 = 0.541$.

We first fit a saturated model permitting unequal variance ratios:

```
TITLE:       EVR not imposed
DATA:        FILE = eqdat.dat;
VARIABLE:        NAMES = x y z;
        USEVARIABLES = x y;
        GROUPING IS z (-1 = cat1 1 = cat2);
MODEL:        fx by x* (sdx1);
        fy by y* (sdy1);
        fx@1;
        fy@1;
        x@0;
        y@0;
        fx with fy;
MODEL cat2:
      fx by x* (sdx2);
      fy by y* (sdy2);
MODEL CONSTRAINT:
      NEW(ratio1, ratio2);
      ratio1 = sdx1^2/sdy1^2;
      ratio2 = sdx2^2/sdy2^2;
```

The output reproduces the sample variance ratios, 0.451 and 0.541.

```
SUMMARY OF ANALYSIS
Number of groups                                                  2
Number of observations
   Group CAT1                                                   500
   Group CAT2                                                   500
Number of dependent variables                                     2
Number of independent variables                                   0
Number of continuous latent variables                             2
Observed dependent variables
  Continuous
   X           Y
Continuous latent variables
   FX          FY
Variables with special functions
  Grouping variable     Z
Estimator                                                        ML
Information matrix                                         OBSERVED
Maximum number of iterations                                   1000
```

```
Convergence criterion                                         0.500D-04
Maximum number of steepest descent iterations                        20
Input data file(s)
  eqdat.dat
Input data format  FREE
THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION
Number of Free Parameters                        10
Loglikelihood
          H0 Value                        -3626.178
          H1 Value                        -3626.178
Information Criteria
          Akaike (AIC)                     7272.356
          Bayesian (BIC)                   7321.433
          Sample-Size Adjusted BIC         7289.673
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                               0.000
          Degrees of Freedom                      0
          P-Value                            0.0000
Chi-Square Contributions From Each Group
          CAT1                                0.000
          CAT2                                0.000
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.000
          90 Percent C.I.                     0.000  0.000
          Probability RMSEA <= .05            0.000
CFI/TLI
          CFI                                 1.000
          TLI                                 1.000
Chi-Square Test of Model Fit for the Baseline Model
          Value                             403.135
          Degrees of Freedom                      2
          P-Value                            0.0000
SRMR (Standardized Root Mean Square Residual)
          Value                               0.000
MODEL RESULTS
                                                       Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Group CAT1
 FX       BY
    X                  0.939      0.030     31.623      0.000
 FY       BY
    Y                  1.398      0.044     31.623      0.000
 FX       WITH
```

```
   FY                 0.737       0.020     36.054      0.000
 Means
   FX                 0.000       0.000    999.000    999.000
   FY                 0.000       0.000    999.000    999.000
 Intercepts
   X                  0.033       0.042      0.774      0.439
   Y                  0.049       0.063      0.784      0.433
 Variances
   FX                 1.000       0.000    999.000    999.000
   FY                 1.000       0.000    999.000    999.000
 Residual Variances
   X                  0.000       0.000    999.000    999.000
   Y                  0.000       0.000    999.000    999.000

Group CAT2
 FX       BY
   X                  2.015       0.064     31.623      0.000
 FY       BY
   Y                  2.739       0.087     31.623      0.000
 FX       WITH
   FY                 0.152       0.044      3.472      0.001
 Means
   FX                -0.020       0.049     -0.410      0.682
   FY                -0.037       0.050     -0.732      0.464
 Intercepts
   X                  0.033       0.042      0.774      0.439
   Y                  0.049       0.063      0.784      0.433
 Variances
   FX                 1.000       0.000    999.000    999.000
   FY                 1.000       0.000    999.000    999.000
 Residual Variances
   X                  0.000       0.000    999.000    999.000
   Y                  0.000       0.000    999.000    999.000
 New/Additional Parameters
   RATIO1             0.451       0.027     16.538      0.000
   RATIO2             0.541       0.048     11.311      0.000
```

The next model restricts the variance ratios to be equal:

```
TITLE:        This is EVR, equal variance ratios
DATA:        FILE = eqdat.dat;
VARIABLE:        NAMES = x y z;
        USEVARIABLES = x y;
        GROUPING IS z (-1 = cat1 1 = cat2);
MODEL:        fx by x* (sdx1);
        fy by y* (sdy1);
        fx@1;
```

```
          fy@1;
          x@0;
          y@0;
          fx with fy;
    MODEL cat2:
          fx by x* (sdx2);
          fy by y* (sdy2);
    MODEL CONSTRAINT:
          NEW(ratio1, ratio2);
          ratio1 = sdx1^2/sdy1^2;
          ratio2 = sdx2^2/sdy2^2;
          ratio1 = ratio2;
```

Its output shows that this model cannot be rejected ($\chi^2(1) = 2.920, p = .0875$), and estimates the variance-ratio to be 0.478.

```
MODEL FIT INFORMATION
Number of Free Parameters                        9
Loglikelihood
          H0 Value                       -3627.638
          H1 Value                       -3626.178
Information Criteria
          Akaike (AIC)                    7273.276
          Bayesian (BIC)                  7317.446
          Sample-Size Adjusted BIC        7288.861
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                              2.920
          Degrees of Freedom                     1
          P-Value                           0.0875
Chi-Square Contributions From Each Group
          CAT1                               0.931
          CAT2                               1.989
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.062
          90 Percent C.I.                    0.000  0.150
          Probability RMSEA <= .05           0.280
CFI/TLI
          CFI                                0.995
          TLI                                0.990
Chi-Square Test of Model Fit for the Baseline Model
          Value                            403.135
          Degrees of Freedom                     2
          P-Value                           0.0000
SRMR (Standardized Root Mean Square Residual)
          Value                              0.031
MODEL RESULTS
```

```
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Group CAT1
 FX       BY
    X                  0.953      0.029     32.672      0.000
 FY       BY
    Y                  1.378      0.042     32.995      0.000
 FX       WITH
    FY                 0.737      0.020     35.987      0.000
 Means
    FX                 0.000      0.000    999.000    999.000
    FY                 0.000      0.000    999.000    999.000
 Intercepts
    X                  0.033      0.043      0.767      0.443
    Y                  0.049      0.062      0.800      0.424
 Variances
    FX                 1.000      0.000    999.000    999.000
    FY                 1.000      0.000    999.000    999.000
 Residual Variances
    X                  0.000      0.000    999.000    999.000
    Y                  0.000      0.000    999.000    999.000

Group CAT2
 FX       BY
    X                  1.955      0.050     39.256      0.000
 FY       BY
    Y                  2.828      0.074     38.129      0.000
 FX       WITH
    FY                 0.151      0.044      3.465      0.001
 Means
    FX                -0.021      0.050     -0.421      0.673
    FY                -0.036      0.050     -0.719      0.472
 Intercepts
    X                  0.033      0.043      0.767      0.443
    Y                  0.049      0.062      0.800      0.424
 Variances
    FX                 1.000      0.000    999.000    999.000
    FY                 1.000      0.000    999.000    999.000
 Residual Variances
    X                  0.000      0.000    999.000    999.000
    Y                  0.000      0.000    999.000    999.000
 New/Additional Parameters
    RATIO1             0.478      0.024     19.996      0.000
    RATIO2             0.478      0.024     19.996      0.000
```

Regression SEMs also can incorporate EVR tests. The variance ratios have to be derived from the error-variances, whence the definitions of `ratio1` and `ratio2` in the syntax below.

```
TITLE:        This is EVR, equal variance ratios
DATA:        FILE = eqdat.dat;
VARIABLE:        NAMES = x y z;
        USEVARIABLES = x y;
        GROUPING IS z (-1 = cat1 1 = cat2);
MODEL:        x* (sdx1);
        y* (se1);
        y on x (b1);
MODEL cat2:
      x* (sdx2);
      y* (se2);
      y on x (b2);
MODEL CONSTRAINT:
      NEW(ratio1, ratio2);
      ratio1 = sdx1/(se1 + (sdx1)*(b1^2));
      ratio2 = sdx2/(se2 + (sdx2)*(b2^2));
      ratio1 = ratio2;
```

This type of model provides estimates of variances rather than standard deviations, which is slightly confusing when comparing its estimates with those of the correlation model. Nonetheless, its estimates are consistent with those of the correlation model. Indeed, the ratio of the estimated correlations in the correlation model and regression coefficients in this model are identical (up to roundoff error): .737/.151 = 0.488 and 1.066/.219 = 0.487.

```
MODEL RESULTS
                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Group CAT1
 Y        ON
    X                  1.066      0.040     26.408      0.000
 Means
    X                  0.033      0.043      0.767      0.443
 Intercepts
    Y                  0.014      0.042      0.347      0.729
 Variances
    X                  0.907      0.056     16.336      0.000
 Residual Variances
    Y                  0.869      0.052     16.682      0.000

Group CAT2
 Y        ON
    X                  0.219      0.063      3.454      0.001
 Means
```

```
   X                   -0.008       0.087      -0.095       0.924
 Intercepts
   Y                   -0.050       0.125      -0.401       0.689
 Variances
   X                    3.822       0.195      19.628       0.000
 Residual Variances
   Y                    7.815       0.410      19.051       0.000
 New/Additional Parameters
   RATIO1               0.478       0.024      19.996       0.000
   RATIO2               0.478       0.024      19.996       0.000
```

SEM Example in Smithson (2012)

The data-set for this example is as described in Smithson (2012), and the file is semex.dat. Starting with SEMs for regression coefficients, we begin with a model that assumes HoV for X but otherwise leaves the other parameters free.

```
TITLE:        HoV in X SEM example
DATA:        FILE = semex.dat;
VARIABLE:        NAMES = x y z;
        USEVARIABLES = x y;
        GROUPING IS z (-1 = cat1 1 = cat2);
MODEL:        x* (sdx1);
        y* (se1);
        y on x (b1);
MODEL cat2:
      x* (sdx2);
      y* (se2);
      y on x (b2);
MODEL CONSTRAINT:
      NEW(ratio1, ratio2);
      ratio1 = sdx1/(se1 + (sdx1)*(b1^2));
  ratio2 = sdx2/(se2 + (sdx2)*(b2^2));
  sdx1 = sdx2;
```

As reported in Smithson (2012), the model fit is $\chi^2(1) = 0.370$ ($p = .543$). Next, we estimate a model that imposes equal variances ratios.

```
DATA:        FILE = semex.dat;
VARIABLE:        NAMES = x y z;
        USEVARIABLES = x y;
        GROUPING IS z (-1 = cat1 1 = cat2);
MODEL:        x* (sdx1);
        y* (se1);
        y on x (b1);
MODEL cat2:
      x* (sdx2);
```

```
      y* (se2);
      y on x (b2);
MODEL CONSTRAINT:
      NEW(ratio1, ratio2);
      ratio1 = sdx1/(se1 + (sdx1)*(b1^2));
  ratio2 = sdx2/(se2 + (sdx2)*(b2^2));
  ratio1 = ratio2;
```

This model yields a chi-square fit of $\chi^2(1) = 82.246$ ($p < .0001$), so we may reject the EVR hypothesis. We then fit a regression model that relaxes EVR but forces equality of regression coefficients and HoV in X.

```
TITLE:       HoV in X and equal slopes SEM example
DATA:        FILE = semex.dat;
VARIABLE:        NAMES = x y z;
        USEVARIABLES = x y;
        GROUPING IS z (-1 = cat1 1 = cat2);
MODEL:        x* (sdx1);
        y* (se1);
        y on x (b1);
MODEL cat2:
      x* (sdx2);
      y* (se2);
      y on x (b2);
MODEL CONSTRAINT:
      sdx1 = sdx2;
      b1 = b2;
```

This model can be rejected because $\chi^2(2) = 15.779$ ($p = .0004$), and the fit comparison between this model and the HoV in X model yields $\chi^2(1) = 15.779 - 0.370 = 15.429$ $p < .0001$), suggesting significantly worse fit. So we conclude that there is moderation of slopes.

Turning now to SEMs for correlations, as before we begin with a model that assumes HoV for X but otherwise leaves the other parameters free.

```
TITLE:       HoV in X SEM example
DATA:        FILE = semex.dat;
VARIABLE:        NAMES = x y z;
        USEVARIABLES = x y;
        GROUPING IS z (-1 = cat1 1 = cat2);
MODEL:        fx by x* (sdx1);
        fy by y* (sdy1);
        fx@1;
        fy@1;
        x@0;
        y@0;
        fx with fy;
```

```
MODEL cat2:
      fy by y* (sdy2);
```

As reported in Smithson (2012), the model fit is $\chi^2(1) = 0.370$ ($p = .543$). Next, a model restricting the variance ratios to equality is estimated:

```
TITLE:       EVR SEM example
DATA:        FILE = semex.dat;
VARIABLE:        NAMES = x y z;
        USEVARIABLES = x y;
        GROUPING IS z (-1 = cat1 1 = cat2);
MODEL:        fx by x* (sdx1);
        fy by y* (sdy1);
        fx@1;
        fy@1;
        x@0;
        y@0;
        fx with fy;
MODEL cat2:
      fx by x* (sdx2);
      fy by y* (sdy2);
MODEL CONSTRAINT:
      NEW(ratio1, ratio2);
      ratio1 = sdx1^2/sdy1^2;
  ratio2 = sdx2^2/sdy2^2;
  ratio1 = ratio2;
```

As with its regression counterpart, this model yields a chi-square fit of $\chi^2(1) = 82.246$ ($p < .0001$). Finally, we estimate an equal-correlation model.

```
TITLE:       HoV in X and equal correlations SEM example
DATA:        FILE = semex.dat;
VARIABLE:        NAMES = x y z;
        USEVARIABLES = x y;
        GROUPING IS z (-1 = cat1 1 = cat2);
MODEL:        fx by x* (sdx1);
        fy by y* (sdy1);
        fx@1;
        fy@1;
        x@0;
        y@0;
        fx with fy (1);
MODEL cat2:
      fy by y* (sdy2);
```

We have $\chi^2(2) = 0.453$ ($p = .797$). The model comparison result is $\chi^2(1) = 0.083$ ($p = .773$), suggesting that the equal-correlations model is a good fit to the data.

Four-Category Moderator

Now we consider a four-category moderator example, with EVR for the first two categories and for the second two, but not for both pairs of categories. We generate an artificial data-set sampled from four bivariate normal distributions, associated with a moderator variable z which takes values 1, 2, 3, and 4, with 300 observations in each category. The variance ratio for the first two moderator categories equals 1/2, whereas for the third and fourth categories the ratio is 1/6. The first two categories' correlations also are identical. The data-set is named fourdat.dat. We begin with a model that tests both EVR hypotheses and the equal-correlations hypothesis.

```
TITLE:       EVR and equal correlations hypotheses: four groups example
DATA:        FILE = fourdat.dat;
VARIABLE:        NAMES = x y group;
        USEVARIABLES = x y;
        GROUPING IS group (1 = cat1 2 = cat2  3 = cat3 4 = cat4);
MODEL:        fx by x* (sdx1);
        fy by y* (sdy1);
        fx@1;
        fy@1;
        x@0;
        y@0;
        fx with fy* (r1);
MODEL cat2:
      fx by x* (sdx2);
      fy by y* (sdy2);
      fx with fy* (r2);
MODEL cat3:
      fx by x* (sdx3);
      fy by y* (sdy3);
      fx with fy* (r3);
MODEL cat4:
      fx by x* (sdx4);
      fy by y* (sdy4);
      fx with fy* (r4);
MODEL CONSTRAINT:
      NEW(ratio1, ratio2, ratio3, ratio4);
      ratio1 = sdx1^2/sdy1^2;
  ratio2 = sdx2^2/sdy2^2;
      ratio3 = sdx3^2/sdy3^2;
      ratio4 = sdx4^2/sdy4^2;
      ratio1 = ratio2;
      ratio3 = ratio4;
      r1 = r2;
```

The results are shown below. The model cannot be rejected ($\chi^2(3) = 2.250, p = .522$), and estimates the variance-ratios to be 0.506 and 0.176.

```
SUMMARY OF ANALYSIS
Number of groups                                                   4
Number of observations
   Group CAT1                                                    300
   Group CAT2                                                    300
   Group CAT3                                                    300
   Group CAT4                                                    300
Number of dependent variables                                      2
Number of independent variables                                    0
Number of continuous latent variables                              2
Observed dependent variables
  Continuous
   X           Y
Continuous latent variables
   FX          FY
Variables with special functions
  Grouping variable     GROUP
Estimator                                                         ML
Information matrix                                          OBSERVED
Maximum number of iterations                                    1000
Convergence criterion                                      0.500D-04
Maximum number of steepest descent iterations                     20
Input data file(s)
  fourdat.dat
Input data format  FREE
THE MODEL ESTIMATION TERMINATED NORMALLY
MODEL FIT INFORMATION
Number of Free Parameters                         17
Loglikelihood
          H0 Value                         -4021.155
          H1 Value                         -4020.031
Information Criteria
          Akaike (AIC)                      8076.311
          Bayesian (BIC)                    8162.842
          Sample-Size Adjusted BIC          8108.844
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                                2.250
          Degrees of Freedom                       3
          P-Value                             0.5222
Chi-Square Contributions From Each Group
          CAT1                                 1.122
          CAT2                                 1.122
```

```
          CAT3                                0.001
          CAT4                                0.004
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.000
          90 Percent C.I.                     0.000  0.087
          Probability RMSEA <= .05            0.764
CFI/TLI
          CFI                                 1.000
          TLI                                 1.001
Chi-Square Test of Model Fit for the Baseline Model
          Value                             904.400
          Degrees of Freedom                      4
          P-Value                            0.0000
SRMR (Standardized Root Mean Square Residual)
          Value                               0.022
MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Group CAT1
 FX       BY
    X                  1.025      0.036     28.517      0.000
 FY       BY
    Y                  1.441      0.050     28.803      0.000
 FX       WITH
    FY                 0.734      0.019     38.935      0.000
 Means
    FX                 0.000      0.000    999.000    999.000
    FY                 0.000      0.000    999.000    999.000
 Intercepts
    X                 -0.040      0.059     -0.671      0.502
    Y                 -0.082      0.083     -0.983      0.325
 Variances
    FX                 1.000      0.000    999.000    999.000
    FY                 1.000      0.000    999.000    999.000
 Residual Variances
    X                  0.000      0.000    999.000    999.000
    Y                  0.000      0.000    999.000    999.000

Group CAT2
 FX       BY
    X                  1.231      0.044     28.015      0.000
 FY       BY
    Y                  1.731      0.062     27.729      0.000
 FX       WITH
    FY                 0.734      0.019     38.935      0.000
 Means
```

```
    FX                 0.031      0.075      0.419      0.675
    FY                 0.064      0.075      0.854      0.393
 Intercepts
    X                 -0.040      0.059     -0.671      0.502
    Y                 -0.082      0.083     -0.983      0.325
 Variances
    FX                 1.000      0.000    999.000    999.000
    FY                 1.000      0.000    999.000    999.000
 Residual Variances
    X                  0.000      0.000    999.000    999.000
    Y                  0.000      0.000    999.000    999.000

Group CAT3
 FX       BY
    X                  1.005      0.040     25.061      0.000
 FY       BY
    Y                  2.395      0.096     25.054      0.000
 FX       WITH
    FY                 0.833      0.018     46.979      0.000
 Means
    FX                 0.045      0.082      0.545      0.586
    FY                 0.090      0.067      1.339      0.180
 Intercepts
    X                 -0.040      0.059     -0.671      0.502
    Y                 -0.082      0.083     -0.983      0.325
 Variances
    FX                 1.000      0.000    999.000    999.000
    FY                 1.000      0.000    999.000    999.000
 Residual Variances
    X                  0.000      0.000    999.000    999.000
    Y                  0.000      0.000    999.000    999.000

Group CAT4
 FX       BY
    X                  1.393      0.049     28.622      0.000
 FY       BY
    Y                  3.321      0.116     28.648      0.000
 FX       WITH
    FY                 0.495      0.044     11.372      0.000
 Means
    FX                 0.051      0.072      0.714      0.475
    FY                 0.110      0.063      1.738      0.082=
 Intercepts
    X                 -0.040      0.059     -0.671      0.502
    Y                 -0.082      0.083     -0.983      0.325
 Variances
```

```
   FX                1.000      0.000    999.000    999.000
   FY                1.000      0.000    999.000    999.000
 Residual Variances
   X                 0.000      0.000    999.000    999.000
   Y                 0.000      0.000    999.000    999.000
 New/Additional Parameters
   RATIO1            0.506      0.028     18.032      0.000
   RATIO2            0.506      0.028     18.032      0.000
   RATIO3            0.176      0.009     18.541      0.000
   RATIO4            0.176      0.009     18.541      0.000
```

We then estimate a model that constrains all variance ratios to be equal. The syntax is shown below. This model can be rejected ($\chi^2(3) = 176.254, p < .0001$).

```
TITLE:        EVR four groups example
DATA:        FILE = fourdat.dat;
VARIABLE:        NAMES = x y group;
        USEVARIABLES = x y;
        GROUPING IS group (1 = cat1 2 = cat2  3 = cat3 4 = cat4);
MODEL:        fx by x* (sdx1);
        fy by y* (sdy1);
        fx@1;
        fy@1;
        x@0;
        y@0;
        fx with fy* (r1);
MODEL cat2:
      fx by x* (sdx2);
      fy by y* (sdy2);
      fx with fy* (r2);
MODEL cat3:
      fx by x* (sdx3);
      fy by y* (sdy3);
      fx with fy* (r3);
MODEL cat4:
      fx by x* (sdx4);
      fy by y* (sdy4);
      fx with fy* (r4);
MODEL CONSTRAINT:
      NEW(ratio1, ratio2, ratio3, ratio4);
      ratio1 = sdx1^2/sdy1^2;
  ratio2 = sdx2^2/sdy2^2;
      ratio3 = sdx3^2/sdy3^2;
      ratio4 = sdx4^2/sdy4^2;
      ratio1 = ratio2;
      ratio1 = ratio3;
      ratio1 = ratio4;
```

As noted earlier, is also possible to incorporate EVR tests in a regression SEM. In this example, the hypothesis that the correlations in the first two categories are identical is equivalent to the hypothesis that the corresponding regression coefficients are identical. The following syntax extracts the EVR hypotheses from the error-variances and variances for X.

```
TITLE:        EVR and equal slopes hypotheses: four groups example
DATA:        FILE = fourdat.dat;
VARIABLE:        NAMES = x y group;
        USEVARIABLES = x y;
        GROUPING IS group (1 = cat1 2 = cat2  3 = cat3 4 = cat4);
MODEL:        x* (sdx1);
        y* (se1);
        y on x* (b1);
MODEL cat2:
        x* (sdx2);
        y* (se2);
        y on x* (b2);
MODEL cat3:
        x* (sdx3);
        y* (se3);
        y on x* (b3);
MODEL cat4:
        x* (sdx4);
        y* (se4);
        y on x* (b4);
MODEL CONSTRAINT:
      NEW(ratio1, ratio2, ratio3, ratio4);
      ratio1 = sdx1/(se1 + sdx1*b1^2);
  ratio2 = sdx2/(se2 + sdx2*b2^2);
      ratio3 = sdx3/(se3 + sdx3*b3^2);
      ratio4 = sdx4/(se4 + sdx4*b4^2);
      ratio1 = ratio2;
      ratio3 = ratio4;
      b1 = b2;
```

The output is shown next. As noted earlier this type of model provides estimates of variances rather than standard deviations, but the estimates are consistent with those in the correlation model. Most importantly, due to the EVR for the third and fourth categories of the moderator, we should expect the ratios of the estimated correlations and regression coefficients in those two groups to be the same, and indeed they are (within roundoff error): $.833/.495 = 1.984/1.181 = 1.68$.

```
MODEL FIT INFORMATION
Number of Free Parameters                     17
Loglikelihood
```

```
          H0 Value                        -4021.155
          H1 Value                        -4020.031
Information Criteria
          Akaike (AIC)                     8076.311
          Bayesian (BIC)                   8162.842
          Sample-Size Adjusted BIC         8108.844
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                               2.250
          Degrees of Freedom                      3
          P-Value                            0.5222
Chi-Square Contributions From Each Group
          CAT1                                1.122
          CAT2                                1.122
          CAT3                                0.002
          CAT4                                0.004
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                            0.000
          90 Percent C.I.                     0.000  0.087
          Probability RMSEA <= .05            0.764
CFI/TLI
          CFI                                 1.000
          TLI                                 1.001
Chi-Square Test of Model Fit for the Baseline Model
          Value                             904.400
          Degrees of Freedom                      4
          P-Value                            0.0000
SRMR (Standardized Root Mean Square Residual)
          Value                               0.022
MODEL RESULTS
                                                     Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Group CAT1
 Y        ON
    X                  1.032      0.039     26.441      0.000
 Means
    X                 -0.040      0.059     -0.670      0.503
 Intercepts
    Y                 -0.041      0.057     -0.722      0.470
 Variances
    X                  1.051      0.074     14.259      0.000
 Residual Variances
    Y                  0.958      0.068     14.007      0.000

Group CAT2
 Y        ON
```

```
   X                  1.032      0.039     26.441      0.000
 Means
   X                 -0.001      0.071     -0.014      0.989
 Intercepts
   Y                  0.030      0.068      0.446      0.655
 Variances
   X                  1.516      0.108     14.008      0.000
 Residual Variances
   Y                  1.382      0.097     14.259      0.000

Group CAT3
 Y        ON
   X                  1.984      0.068     29.129      0.000
 Means
   X                  0.005      0.058      0.093      0.926
 Intercepts
   Y                  0.124      0.077      1.617      0.106
 Variances
   X                  1.010      0.081     12.531      0.000
 Residual Variances
   Y                  1.761      0.141     12.518      0.000

Group CAT4
 Y        ON
   X                  1.181      0.109     10.869      0.000
 Means
   X                  0.032      0.080      0.393      0.695
 Intercepts
   Y                  0.245      0.167      1.469      0.142
 Variances
   X                  1.942      0.136     14.310      0.000
 Residual Variances
   Y                  8.321      0.581     14.327      0.000
 New/Additional Parameters
   RATIO1             0.506      0.028     18.032      0.000
   RATIO2             0.506      0.028     18.032      0.000
   RATIO3             0.176      0.009     18.540      0.000
   RATIO4             0.176      0.009     18.540      0.000
```

References

Muthén, L., & Muthén, B. (2010). Mplus users guide (6th ed.). Los Angeles, CA: Muthén and Muthén.

Smithson, M. (2012). A simple statistic for comparing moderation of slopes and correlations. The Australian National University, Canberra, Australia: Unpublished manuscript.